PROBLEM SET 4

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Exercise 0.1. Let $V \subset \mathbb{A}_k^n$ be an affine algebraic set. Let $I(V/k) := I(V) \cap k[X]$. Show that $V = V(J)$ where for some subset $J \subset k[X]$ if and only if $I(V/k)\bar{k}[X] = I(V)$.

Exercise 0.2. Let $f \in \bar{k}[X_1, \cdots, X_n]$. Show that $V = V(f) \subset \mathbb{A}^n$ is a variety if and only if f is irreducible.

Exercise 0.3. Let $V \subset \mathbb{P}_k^n$ be a projective variety. Let $f : V \to \bar{k}$ be a function such that for all i, $f|_{V \cap U_i} : V \cap U_i \to \bar{k}$ is given by an element in $\bar{k}[V \cap U_i]$. Show that f has to be a constant function.

Exercise 0.4. Let $V = V(Y^2Z - X^3 + X^2Z)$. This is the projective closure of the affine curve $Y^2 = X^3 - X^2$ that is singular at $X = Y = 0$. Let $F = [X/Y : 1] : V \to \mathbb{P}_k^1$. Then F is not regular at $[0 : 0 : 1]$.

Exercise 0.5. Let $V = \mathbb{P}_k^2$. Consider the rational map $F = [X/Y : 1] : V \to \mathbb{P}_k^1$. It is not regular at $p = [0 : 0 : 1]$. Hint: Suppose F is regular at p. Then for any curve C through p, F restricts to a rational map $C \to \mathbb{P}^1$ that is also regular at p, with its value at p the same as $F(p)$. Consider the curves $X = Y^2, Y = X^2$.